PIONEER INTERNATIONAL UNIVERSITY

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UNIVERSITY EXAMINATIONS

END OF SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION

MATH 2112: VECTOR ANALYSIS

DATE:APRIL 2022 TIME: 2 HOURS

INSTRUCTIONS: Attempt Question One and any other Two Questions.

QUESTION ONE (30 MARKS)

a) Find the curl of the vector field $\bar{v} = xyz\, i + yz^2 j + zxy^2\, k$ **[5 marks]**

b) Given that $A = 3i - j + 2k$ and $B = 2i + j - k$, find $A \times B$ **[5 marks]**

c) If $\phi(x, y, z) = 3x^2y - y^3z^2$, find $\nabla\phi$ (or gradϕ) at the point (1,-2,-1) [4 marks]

d) Find the divergence of the vector field given by **[3 marks]**
$\bar{v} = xyz\, i + yz^2 j + zxy^2 \mathrm{k}$

e) Find the value k that will make the vectors $u = (2, 3k, -4, 1, 5)$ and $v = (6, -1, 3, 7, 2k)$ orthogonal **[3 marks]**

f) Given that $A = 2i + 2j - k$ and $B = 6i - 3j + 2k$ Find

(i) the magnitude of **A** **[2 marks]**

(ii) the magnitude of **B** **[2 marks]**

(iii) the dot product of **A** and **B** **[3 marks]**

(iv) the angle between **A** and **B** **[4 marks]**

QUESTION TWO (20 MARKS)

Given the space curve $x = t - \frac{t^3}{3}$, $y = t^2$, $z = t + \frac{t^3}{3}$. Find [(a)]

a) the unit tangent **T** [7 marks]

b) the binormal **B** **[5 marks]**

c) the principal normal **N** **[3 marks]**

d) the curvature κ **[2 marks]**

e) the torsion τ **[3 marks]**

QUESTION THREE (20 MARKS)

If $A = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int_C A \cdot dr$ from (0,0,0) to (1,1,1) along the following paths C: [(a)]

a) $x = t,\ y = t^2,\ z = t^3$ **[5 marks]**

b) the straight lines from (0,0,0) to (1,0,0), then to (1,1,0), then to (1,1,1). [**11 marks**]

c) the straight line joining (0,0,0) to (1,1,1). [**4 marks**]

QUESTION FOUR (20 MARKS)

a) Evaluate $\int\int_S A \cdot ndS$, where $A = 18zi - 12j + 3yk$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant. **[10 marks]**

b) Verify Green's theorem in the plane for
$\oint_C (xy + y^2)dx + x^2dy$
where C is the closed curve of the region bounded by y =x and $y = x^2$. **[10 marks]**

QUESTION FIVE (20 MARKS)

a) Use Stokes' Theorem to evaluate $\iint_S curl\ \vec{F} \cdot d\vec{S}\ where\ \vec{F} = z^2\,\vec{\imath} - 3xy\,\vec{\jmath} + x^3y^3\,\vec{k}$ and S is the part of $z = 5 - x^2 - y^2$ above the plane $z = 1$. Assume that S is oriented upwards. [**10 marks**]

b) Use the divergence theorem to evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = xy\,\vec{\imath} - \frac{1}{2}y^2\,\vec{\jmath} + z\,\vec{k}$ and the surface consists of the three surfaces, $z = 4 - 3x^2 - 3y^2,\ 1 \le z \le 4$ on the top, $x^2 + y^2 = 1,\ 0 \le z \le 1$ on the sides and $z = 0$ on the bottom. [**10 marks**]